

Solution

7	2	7		5		3	0	1
	3		4	9	1	3		7
1	9		4		6		7	2
2		1	0	8		2	8	
6	3		0		5		4	6
	3	7		1	0	6		5
1	8		9		4		6	5
2		9	6	3	1		6	
5	5	0		2		9	7	9

- ◆ 1A: $7997 = 11 \cdot 727 \Rightarrow 727$.
- ◆ 2D: $7887 = 3 \cdot 11 \cdot 239 \Rightarrow 239$.
- ◆ 6A: $1000 < 17^n < 9999 \Rightarrow 4913$.
- ◆ 14D: $1000 < 71^n < 9999 \Rightarrow 5041$.
- ◆ 20D: Only possible digits are $\{6,9\}$ or $\{7,8\}$. Division by 4 $\Rightarrow 96$.
- ◆ 21D $\text{LCM}(23, 29) = 23 \cdot 29 = 667$.
- ◆ 9D: A three digit square such that its middle digit is the unit's digit of its square root.
 $\Rightarrow 9D = 100, 200, 784$ or 900 . But three of them imply that 15A starts with 0. $\Rightarrow 9D = 784$
- ◆ 10A: $3 \mid 10A$ and $2 \cdot 10A$ is a cube $\Rightarrow 2 \cdot 10A = 216$ or $1728 \Rightarrow 10A = 108$ or 864 . By 5D, $10A < 180$ so $10A = 108$.
- ◆ 11A = 28.
- ◆ 8A: $8A < 20\%$ of 20D $\Rightarrow 8A < 20 \Rightarrow 8A = 19$.
- ◆ 4D = "x3" \Rightarrow palindromic $\Rightarrow 33$.
- ◆ 9A = "7x" so by trial and error it is 72.
- ◆ 15A = "4x" so by trial and error it is 46.
- ◆ 25A = "x7x" and palindromic $= \text{mod}(2x-7, 11) = 0 \Rightarrow 2x = 18 \Rightarrow x = 9 \Rightarrow 979$.
- ◆ 4A = "3xy" where $x+y = 1$ and $7 \mid 3xy \Rightarrow "xy" = "01"$ so $4A = 301$.
- ◆ 16D = "6xy" $= 61 \cdot n + 16 \Rightarrow 16D = 655$.
- ◆ 21A = 65 (filled in).
- ◆ 5D: $10A = 108 \Rightarrow 5$ sides. 45 sided polygon $\Rightarrow 5D = 172$ deg.
- ◆ 21A: 7D = "1x", 21A = 65 so by trial and error, 7D = 16 and 12A = 63.
- ◆ 8D = $2 \cdot 12A = 126$.
- ◆ 3D = $\frac{1}{2} \cdot 15A + \frac{2}{7} \cdot 8D = \frac{1}{2} \cdot 46 + \frac{2}{7} \cdot 126 = 23 + 36 = 59$.
- ◆ 18A: by 17A, $18A < 180$. Therefore middle digit = 0 $\Rightarrow 18A = "10x"$ and then 4 factors $\Rightarrow 106$.
- ◆ 17A = $(180 - 18A) / 2 = 74 / 2 = 37$.

- ◆ 22A: By 23D, it is a 4-digit multiple of 301 which ends in 2 \Rightarrow 3612, 6622 and 9632. So 3611, 6621 or 9631. Digits are in descending order \Rightarrow 9631.
- ◆ $23D = (22A+1)/4A + 1 = 9632/301 = 32$.
- ◆ 19A: $8D = 126$ so 2-digit factors are 14, 18, 21, 42 and 63. Also, 19D is a cube so first digit of 19D = 1, 2, 3, 5 or 7. Together, these imply that $19A = 14, 18$ or 21 . However, $19A = 21 \Rightarrow 13D = 331$ which is not $> 4A \cdot 1.1$ so first digit of 19A, 19D is 1. Then, from 22A, 9 and 6 are factors of 19A $\Rightarrow 19A = 18$.
- ◆ $13D = 338$ (filled in).
- ◆ 19D: 3-digit cube, starting with 1 = 125.
- ◆ $22D = 72\%$ of 125 = 90.
- ◆ $6D = "4x0y" =$ multiple of 11 so 6D in {4004, 4103, 4202, 4301, 4400, 4609, 4708, 4807, 4906}. Also $24A = "5z0"$ divides 6D. Together, these imply that $6D = 4400$.
- ◆ and $24A = 550$.